Direct and inverse theorems of rational approximation in Bergman space

For positive numbers $p$ and $\mu$ denote by $A_{p,\mu}$ the Bergman space of analytical functions in a half-plane $\Pi := \{ z \in \mathbb{C} : \text{Im } z > 0 \}$. $R_n(f)_{p,\mu}$ is called the best approximation $f \in A_{p,\mu}$ by rational functions of degree at most $n$. Suppose $\alpha \in \mathbb{R}$ and $\tau > 0$ such that $\alpha + \mu = \frac{1}{p} - \frac{1}{p} > 0$ and $\frac{1}{p} + \mu \notin \mathbb{N}$. We prove that a function $f \in A_{p,\mu}$ satisfies the condition

$$\sum_{n=1}^{\infty} \frac{1}{n} \left( n^{\alpha+\mu} R_n(f)_{p,\mu} \right)^\tau < \infty$$

if and only if $f \in B_\tau^\alpha$, where $B_\tau^\alpha$ is the Besov space of analytic functions in $\Pi$. 

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