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Tractability of Multivariate Integration via Importance Sampling

Importance sampling is a randomized method to improve the performance of numerical algorithms by sampling the really important function values. We investigate how well importance sampling works for the approximation of integrals

$$I(f) = \int_D f(x)\rho(x)\,\mathrm{d}x$$

of functions f in a Hilbert space $H \subset L_1(\rho)$ where ρ is a given probability density on $D \subset \mathbb{R}^d$. We show that whenever the embedding of H into $L_1(\rho)$ is a bounded operator then there exists another density ω such that the worst case error of importance sampling with density function ω is of order $n^{-1/2}$.

This applies in particular to reproducing kernel Hilbert spaces with a nonnegative kernel. As a result, for multivariate problems generated from nonnegative kernels we prove strong polynomial tractability of the integration problem in the randomized setting.

The density function ω is obtained from the application of change of density results used in the geometry of Banach spaces in connection with a theorem of Grothendieck about 2-summing operators.