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Integral estimates for differentiable functions on irregular domains

We set forth new integral representations on irregular domain for any derivative of a function $D^\beta f$ in terms of a set of its derivatives $D^\alpha f$ of the higher orders. They are used to obtain integral estimates of $D^\beta f$ via L_p-norms of corresponding derivatives of function f.

As an example we give the following embedding theorem of Sobolev type.

Theorem. *Let* $G \subset \mathbb{R}^n$ *be a domain with the flexible* σ*-cone property* $(\sigma \geq 1)$, $s, m \in \mathbb{N}$, $1 < p < q < \infty$, $1 \leq r \leq q$, $s - |\beta| - \frac{n}{p} + \frac{n}{q} \geq 0$, $1 \leq m \leq n$, $v(x) = \rho_1(x)^a$, $w(x) = \rho_1(x)^b$, $\rho_1(x) := \min\{1, \operatorname{dist}(x, \partial G)\}$, $b \geq 0$, $a \geq 1 - n - (s-1)(m-1)p$,

$$s - |\beta| - (s-1)(m-1)(\sigma-1) - \frac{\sigma(n-1) + 1 + \sigma a}{p} + \frac{n+b}{q} \geq 0.$$

Then there exists a constant $C > 0$ *such that*

$$\|D^\beta f | L_{q,w}(G)\| \leq C \left[\sum_{j=1}^{m} \sum_{\alpha=\alpha^j,\ |\alpha|=s} \|D^\alpha f | L_{p,v}(G)\| + \|f | L_r(G_\delta)\| \right].$$