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# On global properties of vertical spectra of some hydrophysical characteristics gradients in stratified layers with turbulence patches

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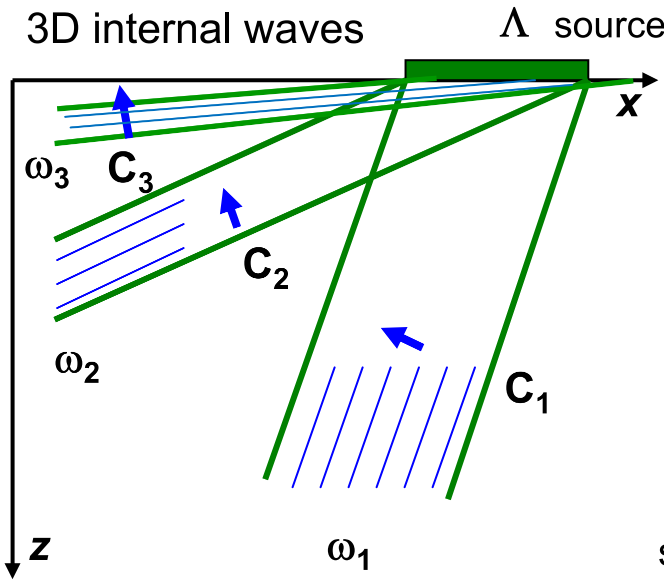
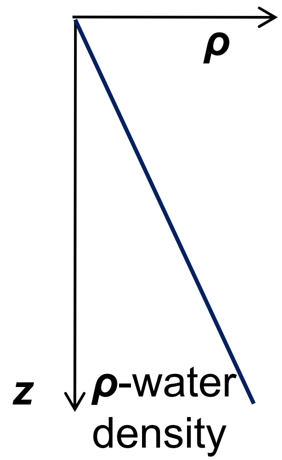
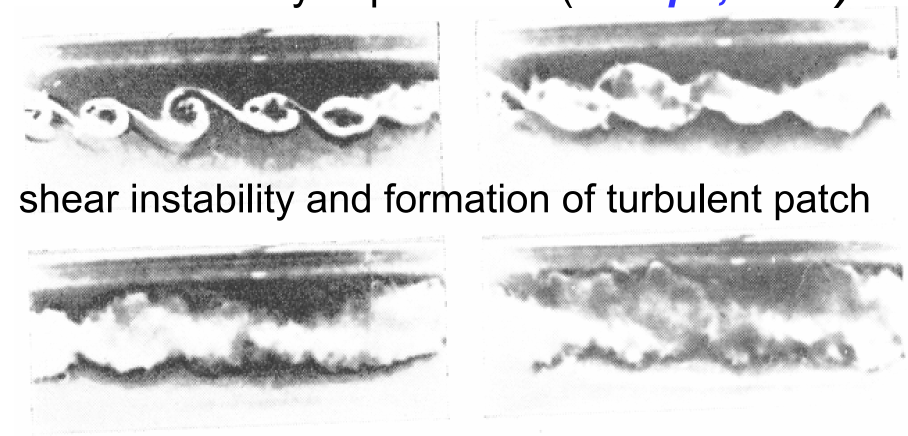
# Overturning of internal waves (**IW**) as a mechanism of turbulent patches formation

3D internal waves



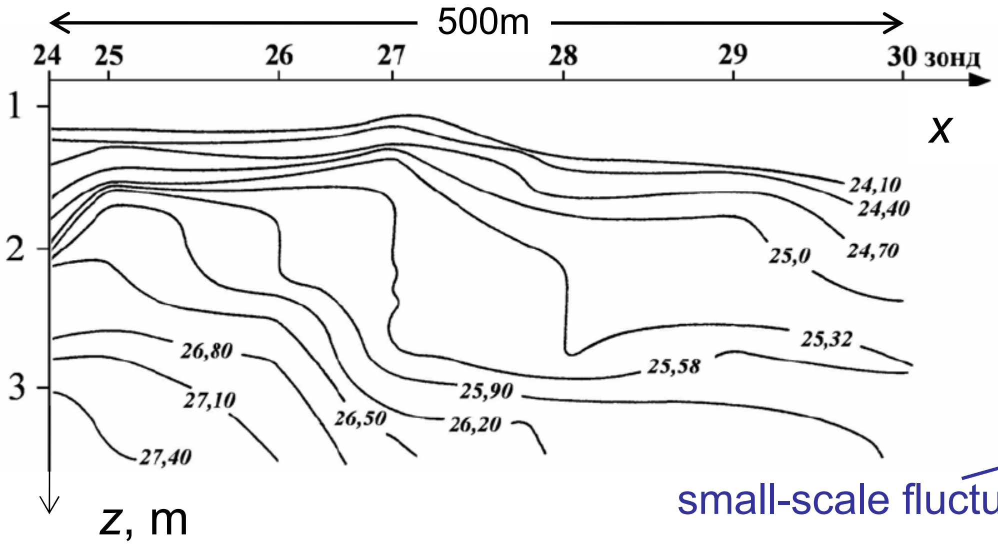

laboratory experiment (*Thorpe, 1971*)



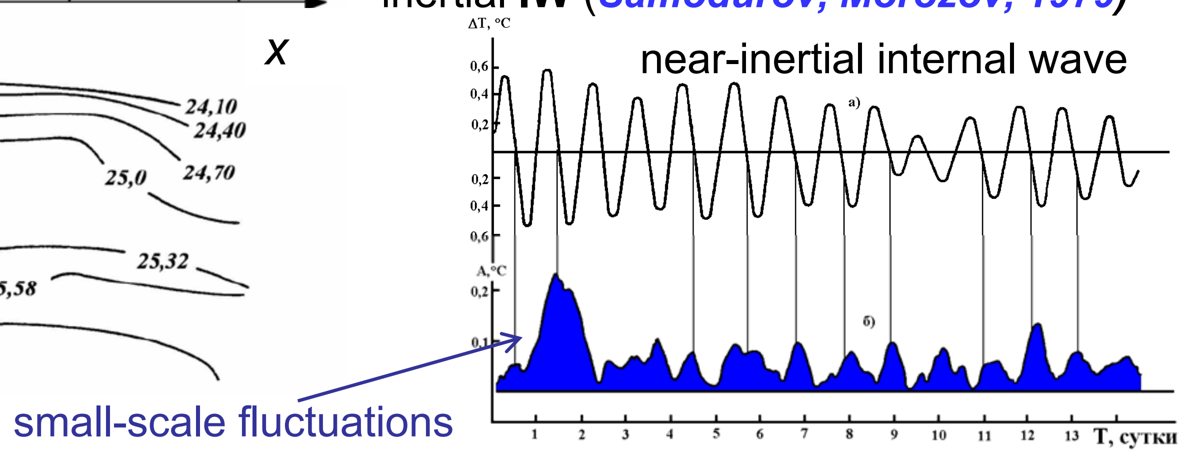

shear instability condition - (*Miles, Howard, 1964*)

$$Ri = N^2 \Big/ \left(\frac{\partial U}{\partial z}\right)^2 < \frac{1}{4}, \quad N = \sqrt{\frac{g}{\rho}\frac{\partial \rho}{\partial z}}$$

Natural turbulent patches, (*Samodurov et al., 1991*)




Instability events at a certain phase of near-inertial **IW** (*Samodurov, Morozov, 1979*)

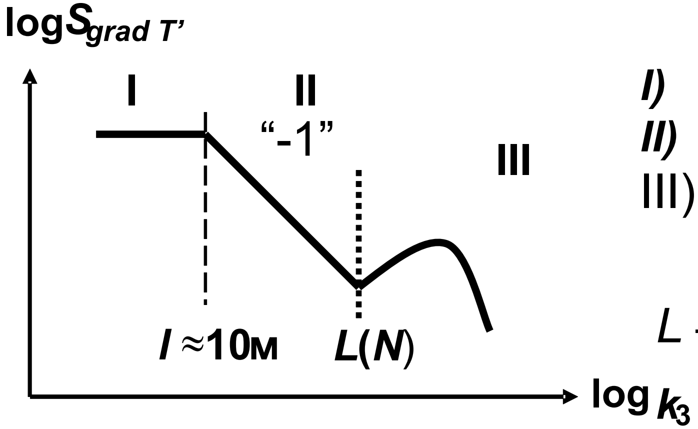


# Effective scale of turbulent patches estimate based on vertical profiling of stratified layers in natural basins



Fields under consideration: vertical fluctuations of temperature, salinity and horizontal velocity and their vertical gradients

a typical vertical gradients spectrum scheme (***Gregg, 1977***)



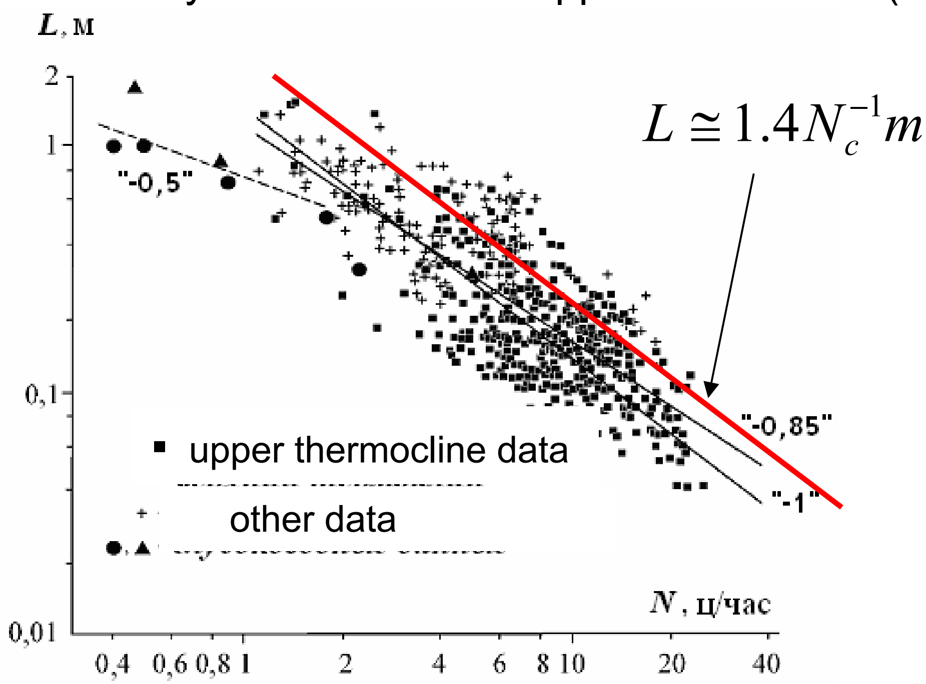

*I)* ***IW*** without energy loss for turbulent dissipation

*II)* ***IW*** that loss a part of energy for turbulent dissipation

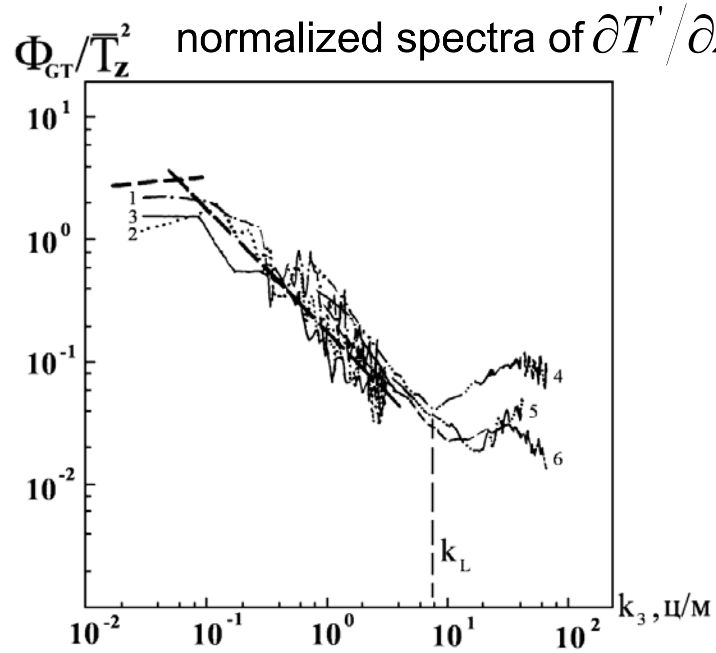
III) "Turbulent energy" within patches at the expense of ***IW*** breaking

*L* – effective scale (thickness) of turbulent patches

Data analysis for the ocean upper thermocline (more than 400 spectra), ***Samodurov et al., 1994***




normalized spectra of $\partial T' / \partial z$



# General methodology of modeling

## Main purpose:

construction of the complete model of the turbulent exchange in stratified layers in ocean

1. Mathematical model of turbulence in patch based on Kolmogorov hypothesis and experimental data.
2. Modeling of patches distribution by size and location in the ocean stratified layer.
3. Verification of this models by numerical simulations and comparison with real spectra of turbulence.

## Density in patch

We suggest that density depend on patch scale as

$$\rho(z) = L\,\psi\left(\frac{z-\tau+L/2}{L}\right) + c, \quad z \in \Delta,$$

$$\Delta = \left[\tau - L/2, \tau + L/2\right]$$

$L$ – effective scale of turbulent patches,

$\psi$ – function with compact support in [0,1]

$\tau$ is center of patch, $c$ - constant

## Multifractal decomposition

In accordance to ***Jaffard et al.,2001*** we can assume that $\psi(z)$ in "unit" patch $[0,1]$ is a combination of ***chirps***:

$$f_{\alpha,\beta,z_0}(z) = A\left|z - z_0\right|^{\alpha} \cos\left(B\left|z - z_0\right|^{-\beta}\right), \quad \alpha, \beta > 0, \; z_0 \in (0,1)$$

$\alpha$ – Hölder exponent, $\beta$ – oscillation exponent

The Hausdorff dimension of the set of $z_0$, where chirps with $\alpha$, $\beta$ exist, one has (***Jaffard et al.,2001*** )

$$D(\alpha,\beta) = 1 - \inf_{p,s}\left\{(1+\beta)s + p\alpha + \eta(p,s)\right\}$$

$$\eta(p,s) \approx \frac{\ln\left(\int_R d^p_{s/p}(a,b)\,db\right)}{\ln a}\ , \quad d_s(a,b) = \sup_{\substack{0<x\le a \\ b-a\le y\le a+b}} \left|x^s W(\psi,x,y)\right|$$

$W(\psi,x,y)$ – wavelet transform

Spectrum of chirp (***Innocent, Torresani, 1996***):

$$\widehat{f}_{\alpha,\beta,z_0}(\nu) \approx C(\alpha,\beta,A,B)\,\nu^{-\frac{2\alpha+\beta+2}{2(\beta+1)}} e^{i\eta(\nu;\alpha,\beta,A,B)}$$

$C(\alpha,\beta,A,B)$ – known constant

$\eta(\nu;\alpha,\beta,A,B)$ – known function of variable $\nu$

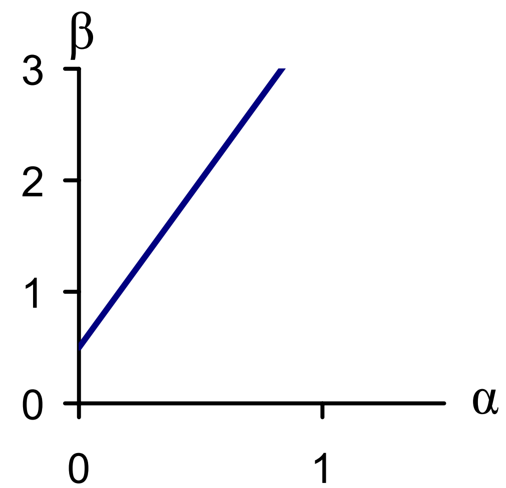
$$|\widehat{f}_{\alpha,\beta,z_0}(\nu)|^2 \sim \nu^{-\frac{2\alpha+\beta+2}{\beta+1}}$$ – evaluation of spectrum slope

In accordance to (***Runovski, Schmeisser, 2004)*** we can localize the chirp by multiplying it on infinitely differentiable function $\varphi(z-z_0)$ with compact support without affecting the spectrum form

# The structure of $\psi(z)$ and discrete model

By Kolmogorov' theory validated experimentally the exponents in chirps are connected as follows

$$\frac{2\alpha+\beta+2}{\beta+1}=\frac{5}{3} \quad \Longrightarrow \quad \beta = 3\alpha + \frac{1}{2}$$




For discrete model

$$\alpha_j = \delta j\,,\ \ j=1,2,\ldots,J\,. \quad \beta_j = 3\alpha_j + 1/2$$

For the set $\Omega_j$ of points $z_0$ its Hausdorff dimension is denoted by $d_j$

$N_j = [a^{d_j}],\quad j=1,2,\ldots,J,$ – number of centers $\quad \sum_{j=1}^{J} a^{d_j} \approx N$ – total number of chirps

Together with ***homogeneity*** hypothesis of Kolmogorov we consider ***uniformity*** hypothesis. It implies that all *A* and *B* in chirps are statistically identical in $\Omega_j$

Finally, we obtain the following formula

$$\psi(z) = \sum_{j=1}^{J}\sum_{k_j=1}^{N_j} A_j\,|\,z-\xi_j^{k_j}\,|^{\alpha_j}\,\cos\left(B_j\,|\,z-\xi_j^{k_j}\,|^{-(3\alpha_j+1/2)}\right)\varphi(z-\xi_j^{k_j})$$

$\xi_j^{k_j}$ - finite array of points in [0,1] for $\Omega_j$

# Numerical simulation

## Problems

- Modeling a turbulent process in patch
- Modeling a turbulent spectra in layer with many patches
- Modeling other dynamic processes (internal waves)
- Model validation (composed turbulence and internal waves spectra) on experimental data

Now we can use only well known Monin-Obukhov model spectrum and empirical spectrum of Nasmith ***(Nasmith,1970***)

$E(k) = C\varepsilon^{2/3}k^{-5/3}$ – Monin-Obukhov spectrum of turbulence

$\varepsilon = \mu \left( \frac{dv_i'}{dx_k} \right)^2$ – the rate of turbulent energy dissipation

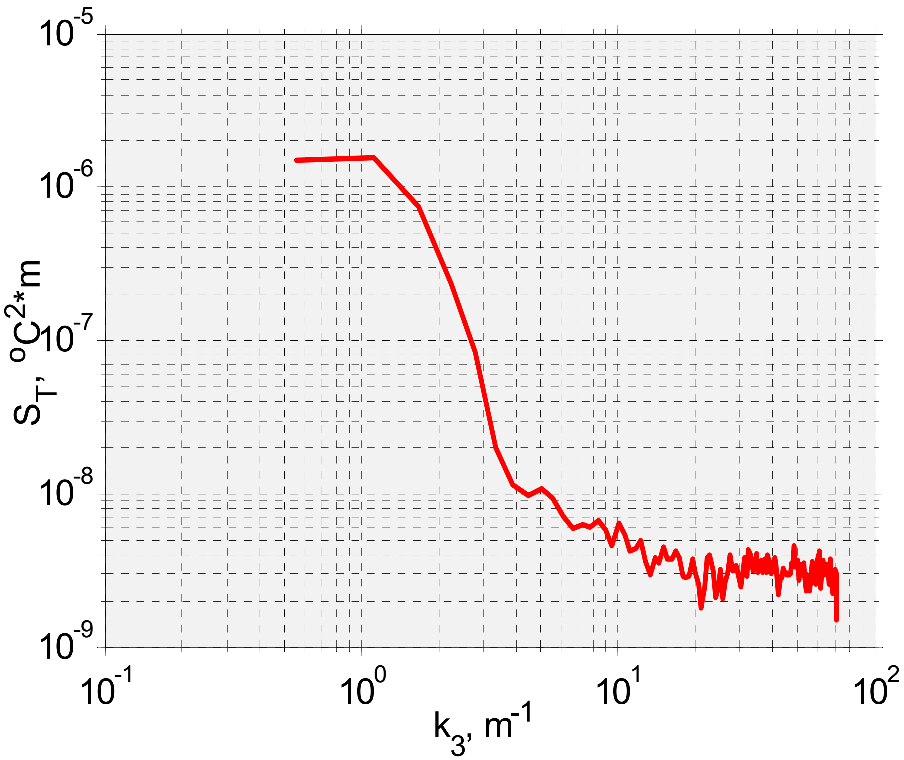
$k = 2\pi/l$ – wave number, $\mu$ – viscosity

Our goal is obtaining a model spectrum that correspond to real spectra in ocean.

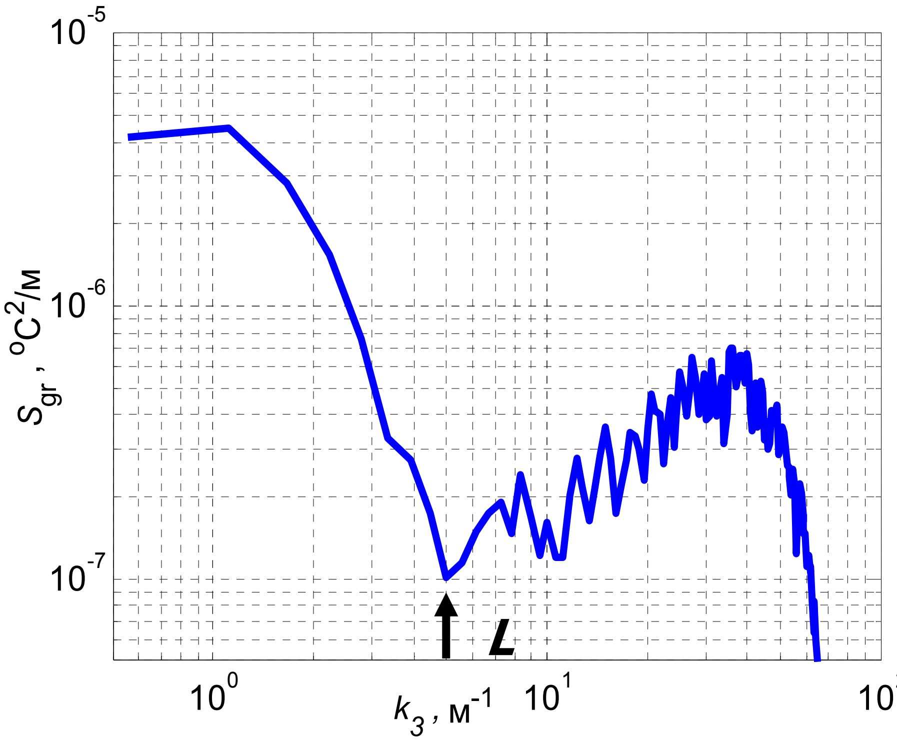
# Real spectra of hydrophysical characteristics in stratified layer

(*Samodurov, Chukharev, 2008*)

**Spectrum of temperature fluctuations**




**Spectrum of temperature gradients**



# Steps in modeling

## Step 1. Specifying and determining parameters for mathematical model:

$N$ – total number of chirps

$\delta$ - subinterval of axis $\alpha$

$\varepsilon$ – "½" of the support of $\varphi(z)$

$J$ – number of exponents in model

$d_j$ – Hausdorff dimensions (from experimental data)

$\alpha_j = \delta j, \; j = 1, 2, \ldots, J,$ – number of oscillation exponents

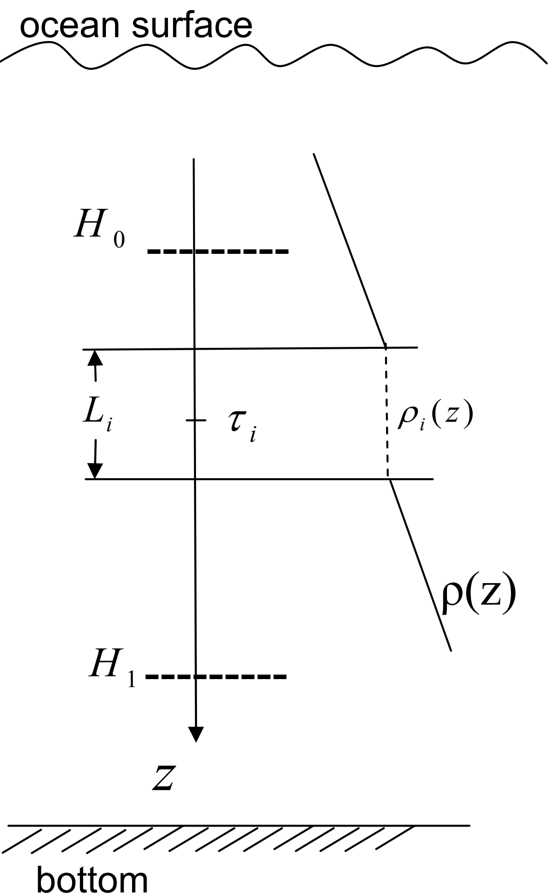
$\Lambda_j = \{\xi_j^{k_j}\}_{k_j=1}^{N_j}$ - uniformly distributed on $[\varepsilon, 1\text{-}\varepsilon]$ random values

## Step 2. Choice of parameters *A* and *B* for chirps

Initially: $B_1 = \ldots = B_J = 1, \quad A_1 = \ldots = A_J = A$ In order to satisfy $\|\psi\|_{C[0,1]} \ll c_0$

where $c_0 = \max\{c/L\}$ we require $A \ll c_0 \left( \sum_{j=1}^{J} N_j \varepsilon^{\alpha_j} \right)^{-1}$

Step 3. Modeling the distribution of physical parameters in layer with $m$ patches




Let $L_1 \geq \ldots \geq L_m$

Then

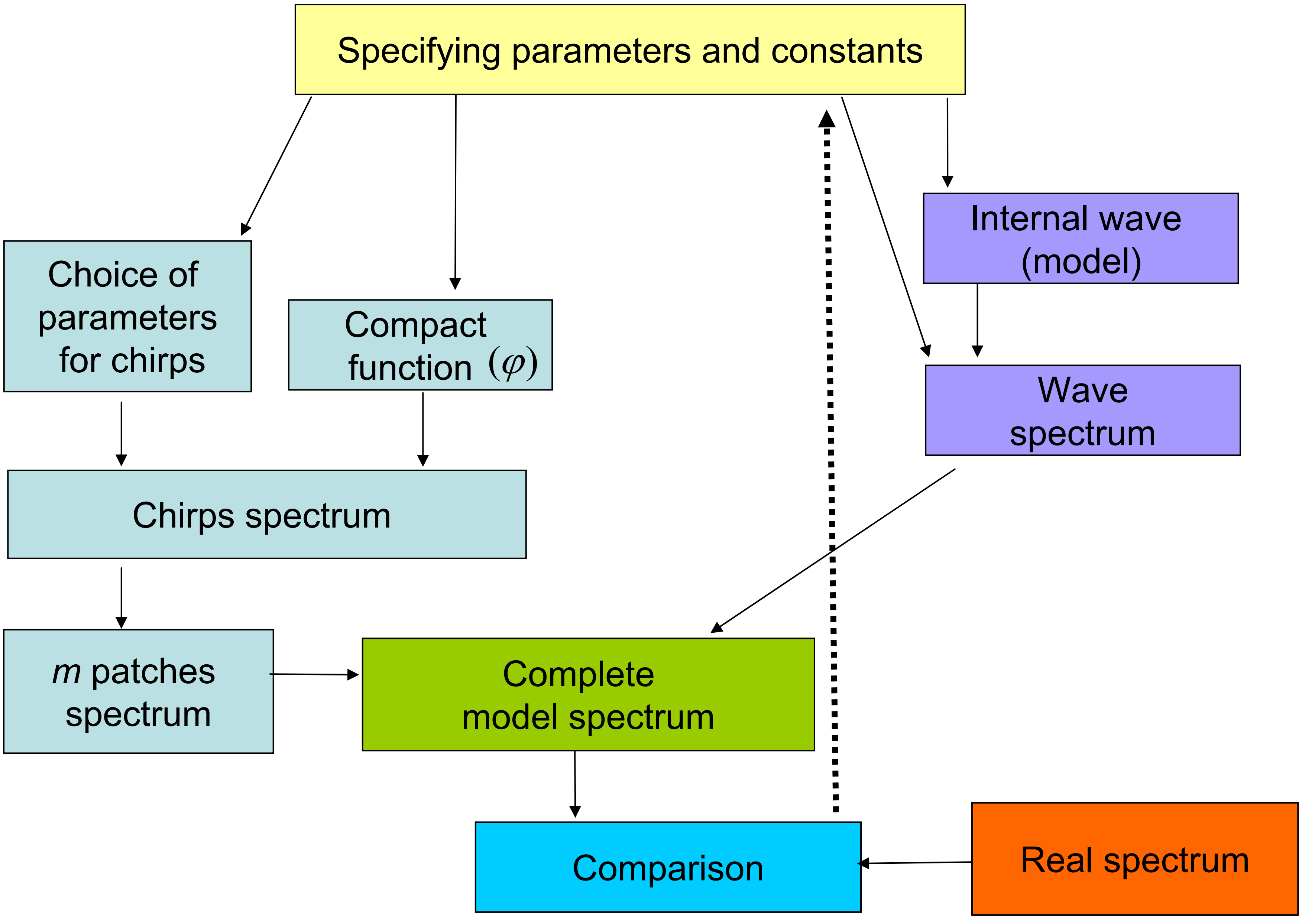
$$\rho(z) = \rho_w(z) + \sum_{i=1}^{m} \rho_i(z)$$

$\rho_w$ – wave component

Step 4. Modeling of internal waves and its spectrum

Step 5. Multiplexing of turbulence model and waves model

# Program block diagram



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